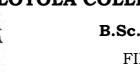
LOYOLA COLLEGE (AUTONOMOUS), CHENNAI - 600 034



B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIRST SEMESTER - NOVEMBER 2011

MT 1501 - GRAPHS, DIFF. EQU., MATRICES & FOURIER SERIES

Date: 10-11-2011 Time: 1:00 - 4:00 Dept. No.

Max.: 100 Marks

Part A $(10 \times 2 = 20)$

Answer All questions.

- 1. Find the equation of the line passing through (2, 9) and (2, -9).
- 2. Find the axis and vertex of the parabola $y = x^2 + 2x + 3$.
- 3. Write the normal equations of y = ax + b.
- 4. Define linear law.
- 5. Form the difference equation of lowest order by eliminating the arbitrary constants a and b from $y = (a + bx) 2^x$.
- 6. Solve $y_{n+2} y_{n+1} + y_n = 0$.
- 7. Define symmetric and skew-symmetric matrices.
- 8. Find the eigen values of the matrix $\begin{pmatrix} a & h & g \\ 0 & b & 0 \\ 0 & 0 & c \end{pmatrix}.$
- 9. Find the Fourier coefficient a_n for the function $f(x) = e^x$ in $(-\pi, \pi)$
- 10. Define odd and even functions.

Part B $(5 \times 8 = 40)$

Answer any Five questions.

- 11. The total cost (in rupees) of output x is given by $C = \frac{2}{3}x + \frac{35}{2}$. Find
 - (i) The cost when the output is 4 units.
 - (ii) The average cost of output of 10 units.
 - (iii) The marginal cost when the output is 3 units.
- 12. A firm produces x units of output per week at a total cost of Rs. $\frac{1}{3}x^3 x^2 + 5x + 3$. Find the value of x at which the marginal cost and the average cost attain their respective minimum.
- 13. Using the method of least squares, fit a straight line to the following data.

X	5	10	15	20	25
у	15	10	20	26	30

- 14. Explain the method of least squares.
- 15. Solve the difference equation $y_{n+2} 2 y_n \cos \alpha + y_{n-1} = 0$. If $y_0 = 0$ and $y_1 = 1$, Find y_2, y_3, y_4 .
- 16. Find the eigen values and eigen vectors of $\begin{pmatrix} 1 & 2 & 3 \\ 0 & 2 & 3 \\ 0 & 0 & 2 \end{pmatrix}$
- 17. Verify Cayley Hamilton theorem for the matrix $\begin{pmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{pmatrix}$
- 18. Obtain the Fourier expansion for $f(x) = x \pi$ in the interval $(-\pi, \pi)$

Part C
$$(2 \times 20 = 40)$$

Answer any Two questions.

19. (a) From the table given below, fit an equation of the form $y = a + bx + cx^2$.

X	87.5	84	77.8	63.7	46.7	36.9
y	292	283	270	235	197	181

(b) The price and demand for an item are related by $p = 32 - x^2$, while price and supply are related by $p = x^2$. Draw the graph and find the equilibrium supply and equilibrium price.

$$(12 + 8)$$

20. Solve the difference equations:

(a)
$$y_{x+2} - 5y_{x+1} + 6y_x = x^2 + x + 1$$

(b)
$$u_{n+2} - 7 u_{n+1} - 8u_n = 2^n n^2$$
 (10+10)

- 21. (a) Obtain the half range cosine series for f(x) = x in $(0, \pi)$ and deduce that the sum of the series $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \frac{\pi^2}{8}$
 - (b) Find a Fourier series expansion for the function $f(x) = \begin{cases} -1, -\pi < x < 0 \\ 1, 0 \le x \le \pi \end{cases}$

$$(10 + 10)$$

22. Diagonalize the matrix $\begin{pmatrix} 2 & -2 & 3 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{pmatrix}.$
